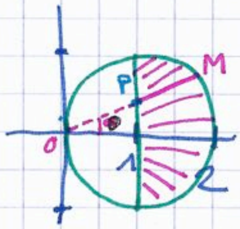
(3) $I=\iint \frac{dx\,dy}{x}$

$D\begin{cases} x\geq 1 \\ x^2+y^2-2x\leq 0 \end{cases} \iff \begin{cases} x\geq 1 \\ (x-1)^2+y^2\leq 1 \end{cases}$

$I=\int_{\theta=-\frac{\pi}{4}}^{\pi/4}\int_{r=\frac{1}{\cos\theta}}^{2\cos\theta} \frac{\cancel{r}\,dr\,d\theta}{\cancel{r}\cos\theta}$

constant par rapport à r



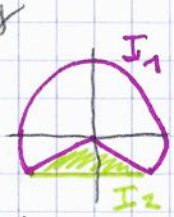
OM = 2 cos α

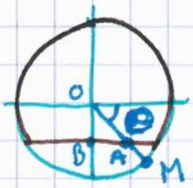
OP = $\frac{1}{\cos\alpha}$

$I=\int_{\theta=-\frac{\pi}{4}}^{\pi/4}\frac{1}{\cos\theta}\times\left(2\cos\theta-\frac{1}{\cos\theta}\right)d\theta = \int_{-\pi/4}^{\pi/4}\left(2-\frac{1}{\cos^2\theta}\right)d\theta$

$=2\frac{\pi}{2}-[\tan\theta]_{-\pi/4}^{\pi/4} = \pi-2$

(6) $\iint (x^2+y^2)\,dx\,dy$





$OA=\frac{OB}{\sin\theta}=\frac{1}{2\sin\theta}$

$I_1=\int_{\theta=-\pi/6}^{7\pi/6}\int_{r=0}^{1} r^2\,r\,dr\,d\theta$ et $I_2=\int_{\theta=-5\pi/6}^{-\pi/6}\int_{r=0}^{\frac{1}{2\sin\theta}} r^2 r\,dr\,d\theta$

$I_1=\int_{\theta=-\frac{\pi}{6}}^{7\pi/6} d\theta\int_{r=0}^{1} r^3\,dr$

$=\frac{4\pi}{3}\times\frac{1}{4}=\frac{\pi}{3}$

$I_2=\int_{-5\pi/6}^{-\pi/6}\frac{1}{(2\sin\theta)^4}.\,d\theta$

→ Bioche $x=\tan\theta$ pose des pb aux bornes alors : $x=\text{cotan}\,\theta$, $dx=\frac{d\theta}{\sin^2\theta}$

$=2\int_{-\frac{\pi}{2}}^{-\pi/6}\frac{1}{16}\frac{d\theta}{\sin^4\theta}$

il manque fois 1/4

$=\frac{1}{8}\int_0^{-\sqrt{3}} -dx\times(1+x^2)$

$=-\frac{1}{8}\left[x+\frac{x^3}{3}\right]_0^{-\sqrt{3}}$

$=\frac{1}{8}\left(\sqrt{3}+\frac{3\sqrt{3}}{3}\right)=\frac{\sqrt{3}}{4}$ *donc ici √3/16*

$I=\frac{\pi}{3}+\frac{\sqrt{3}}{16}$

Nota : si $T=\tan\theta$ et $K=\text{cotan}\,\theta$ alors, en posant $S=\sin\theta$ et $C=\cos\theta$:

$T=\frac{S}{C}$, $T'=\frac{1}{C^2}=1+T^2$ | $K=\frac{C}{S}$, $K'=\frac{-1}{S^2}=-1-K^2$

le (5) feuille suivante